

A comparison of P code and high performance C/A code GPS receivers for on the fly ambiguity resolution

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ABSTRACT. Carrier phase ambiguity resolution on the fly is investigated using two receiver technologies, namely dual-frequency P code and high performance, single frequency, C/A code receivers. Both receiver types were used simultaneously in a series of land kinematic trials. A least-squares search technique is used to find the correct double difference carrier phase ambiguities. Both C/A and single frequency P code technologies are found to be equivalent and capable of resolving the integer ambiguities on the fly using some 30 to 200 seconds of data under benign multipath conditions. Successful ambiguity resolution on the fly results in cm-level accuracy kinematic positioning. The ambiguity resolution time required and success rate are however found to be strongly dependent on the level of carrier phase multipath and, as a consequence, on the error variance assigned to the carrier phase measurements. The use of widelaning with the dual frequency P code results in ambiguity resolution in seconds. The performance of widelaning is also superior in a comparatively high carrier phase multipath environment.

INTRODUCTION

Carrier phase ambiguity resolution on the fly, i.e., without static initialization, is the solution to sub-decimetre kinematic positioning without the operational constraint of static initialization as used in semi-kinematic or stop and go positioning (e.g., Cannon 1990). Various methods are available to resolve the ambiguities, e.g., the ambiguity function method (Mader 1990), the least-squares search approach (Hatch 1991, Remondi 1991), and the Fast Ambiguity Resolution Approach (Frei & Beutler 1990). While some of these methods are mathematically equivalent, e.g., the ambiguity function method and the least-squares search approach (Lachapelle et al 1991, 1992), different implementation schemes may result in specific advantages and/or disadvantages (e.g., Erickson 1992). For instance, the sequential square root information filtering

method proposed by Landau and Euler (1992) results in substantially lower computational requirements. The level of success of any technique is a function of several parameters, namely the type and quality of observables used, the multipath environment, the distance between the monitor and mobile receivers, the number of satellites available and their geometry. A larger number of satellites and a shorter distance between the monitor and mobile will result in relatively faster ambiguity resolution due to the higher measurement redundancy and the lower differential atmospheric and orbital errors.

The objective of this paper is to compare the performance of two receiver technologies for ambiguity resolution on the fly for land kinematic applications. These two technologies consist of high performance single frequency C/A code and dual frequency P code receiver technologies. High performance C/A code receivers deliver C/A code measurements with a level of accuracy similar to that of P code L 1 measurements.

EQUIPMENT SELECTION

The high performance C/A code receiver technology selected for the test is the GPSCard™, designed and built by NovAtel Communications Ltd. The P code receiver selected is the 36-channel Ashtech P-XII receiver. The GPSCard™ is a high performance 10-channel C/A code unit which has two unique characteristics, namely a 10-cm code noise and a narrow code spacing option to reduce code multipath interference (Fenton et al 1991, Erickson et al 1991, Cannon & Lachapelle 1992a, Van Dierendonck et al 1992). Earlier ambiguity resolution on the fly tests with the GPSCard™ confirmed its capability (Cannon et al 1992). The P-XII is a 36 channel unit which measures carrier phase and C/A, P(L1), and P(L2) code simultaneously. The C/A code noise of the P-XII is 100 cm while the corresponding P code noise is 10 cm. The phase noise is stated as 0.2 mm for the P-XII and 0.4 mm (diff. channel) for the GPSCard™.

In practice, receiver noise and multipath cannot be separated and their combined effect becomes the prime error source in the resolution of the phase ambiguities on the fly. The estimated combined rms effect of receiver code noise and multipath for both receivers is given in Table 1. These figures can be considered pessimistic for the case of a normal kinematic environment since they were derived from a series of static experiments in a high multipath environment (Cannon & Lachapelle 1992b). The narrow correlator spacing on the code tracking loops of the GPSCard™ results in a combined rms effect of 70 cm, the same figure as that of the P-XII P code. The corresponding P-XII C/A code rms effect of 300 cm is at the level anticipated from standard C/A technology. In both cases, the use of chokering ground planes results in a significant multipath reduction. Carrier phase multipath is also significant at the cm level. In kinematic mode, the combined receiver noise and multipath can vary from a few mm to a few cm. The impact of this effect on phase ambiguity resolution on the fly turns out to be very significant as will be shown later.

Table 1
GPSCard™ and P-XII Code Measurement Characteristics in a High Multipath Environment

Receiver & Obs. Type	Measuring Noise (rms)	Noise + Multipath (rms)
GPSCard™	10 cm	70 cm
GPSCard™ with chokering		30 cm
P-XII C/A code	100 cm	300 cm
P-XII C/A code with chokering		200 cm
P-XII P code	10 cm	70 cm
P-XII P code with chokering		30 cm

AMBIGUITY RESOLUTION METHODOLOGY

The procedure used herein for carrier phase ambiguity resolution on the fly involves two steps. Firstly, differential code measurements are used to estimate an approximate position for the kinematic antenna to limit the potential number of integer carrier phase ambiguity solutions. A more accurate code solution limits the number of potential solutions which results in faster convergence to the correct solution. If the correct solution is outside the initial search volume defined by the code solution, it will evidently never be found and the algorithm will fail. The accuracy of the differential code solution therefore plays an important role in improving processing

efficiency. Secondly, a least-squares search technique is used to isolate the correct integer ambiguity combination.

Two properties of the least-squares search technique are used, namely (i) only three of the double difference carrier phase ambiguities are independent, and (ii) the estimated variance factor calculated using the adjusted carrier phase residuals should be minimum at the correct solution. The first property means that once three double difference phase ambiguities are known, the position of the moving receiver can be precisely determined, and the ambiguities of the remaining satellites can be fixed. Four primary satellites are needed to generate an entire set of potential solutions which are computed based on different trials of double difference carrier phase ambiguities. Each potential solution, which corresponds to one specific three-ambiguity set of the primary satellites, is checked using observations from the redundant or secondary satellites. At the potentially correct solution, the computed observations for the secondary satellites should be very close to the corresponding measured observations, i.e. the residuals should be minimum. This implies that the estimated variance factor should also be a minimum. The estimated variance factor is computed as

$$\hat{\sigma}_0^2 = \frac{\mathbf{v}^T \mathbf{C}^{-1} \mathbf{v}}{n-u} \quad (1)$$

where $\hat{\sigma}_0^2$ is the estimated variance factor, \mathbf{v} is the vector of measurement residuals, \mathbf{C}^{-1} is the measurement covariance matrix, n is the number of double difference observations and u is the number of unknown parameters which is three in this case. For each integer ambiguity solution, the estimated variance factor is compared to the *a priori* variance factor using a Chi-square test. If the test fails for a particular integer ambiguity combination, this potential solution is rejected. Clearly from Eq. (1), the magnitude of the estimated variance factor is a function of the measurement accuracy, i.e. \mathbf{C}^{-1} . For the double difference carrier phase measurement case, the combined effect of receiver noise and multipath must be taken into account.

If more than one potential solution are passed through the agreement test at a certain epoch, the ambiguity sets corresponding to these potential solutions are retained and further tested at the following epochs. In this case, a variance factor based on the previous carrier phase measurements is tested. This is called 'global' variance factor testing, since more than one epoch is considered. As more epochs are used, all the false ambiguity sets of the **primary** satellites will gradually be rejected except the correct one. The more satellites available, the less the observation time required for resolving the ambiguities. For more information on statistical testing criterion for on the fly ambiguity resolution, see Lachapelle et al. (1991).

In order to accelerate the convergence time and reduce the effect of a possible bias on the *a priori* carrier phase variance ($\sigma^2 \nabla \Delta \phi$), a ratio test is also used. When the

number of potential solutions is reduced to a relatively low number after global testing, the ratio of the two **smallest variance factors is computed. If the ratio is greater than a preset value (2 was used for the results presented in this paper), the potential ambiguity solution with the smaller variance is selected as the correct ambiguity set.**

FIELD TEST

A series of semikinematic, i.e., stop and go, tests were conducted in August 1992 on the portion of the Springbank Baseline shown in Figure 1. The Springbank baseline is located some 20 km west of Calgary. The test made on August 25 was used to derive the results presented in this paper. Only the kinematic portion of the test was used to investigate carrier phase ambiguity resolution on the fly. The control points were used to independently determine the ambiguities using a static initialization method (e.g., Cannon 1990) and to provide a reference trajectory at the cm-level for verifying the quality of the on the fly ambiguity solutions. The test was conducted in differential mode and two P-XII and two GPSCard™ units were used simultaneously. The two monitor stations (i.e. one P-XII and one GPSCard™) were set up at two different stations, as shown in Figure 1. The distance between any one monitor station and the vehicle was less than 3 km. The two receiver antennas mounted on the vehicle roof were separated by some 90 cm. A medium tracking bandwidth of 5 Hz was used for the GPSCard™ units. Choking ground planes were used with all four units to minimize the effects of multipath as shown in Table 1. The data was collected at a rate of 1 Hz.

Figure 1 shows the presence of trees near the corner of the L-shape trajectory. This was done deliberately to analyze the effect of multipath caused by nearby trees. These trees did not, however, result in signal masking or loss of phase lock. The Signal-to-Noise Ratio (SNR) measured by the GPSCard™ units on SV28 and SV17 during the test are shown in Figure 2 and 3. The portion of the trajectory affected by trees was observed between 260500 s and 260700 s. There is no relative drop of SNR at the remote unit during this period. Likewise, no SNR drop was observed by the P-XII unit during that section of the trajectory.

Seven satellites with an elevation $2 \cdot 10''$ were observed during the test with a GDOP ≤ 3 . The vehicle speed ranged from 50 to 70 km/h.

ANALYSIS OF RESULTS

In order to assess the quality of the data before attempting to resolve the ambiguities on the fly, the kinematic data was processed using a standard static initialization procedure. The carrier phase residuals were analyzed to quantify the combined effect of carrier phase measuring noise and multipath. Representative carrier phase residuals for SV1 and 28 are shown in Figure 4.5 and 6 for the P-

XII P2, P-XII (P1 - P2) and GPSCard™ units, respectively. The corresponding P-XII P1 phase residuals exhibit trends similar to the P2 residuals shown in Figure 4. The residuals are below the 5- 10 mm level during the clear portion of the trajectory. However, during the corner portion where trees create multipath, the residuals increase substantially. This will affect significantly the ambiguity resolution on the fly performance as described below. The combined effect of code noise and multipath ranged from 15 to 35 cm which is consistent with the value of 30 cm quoted in Table 1.

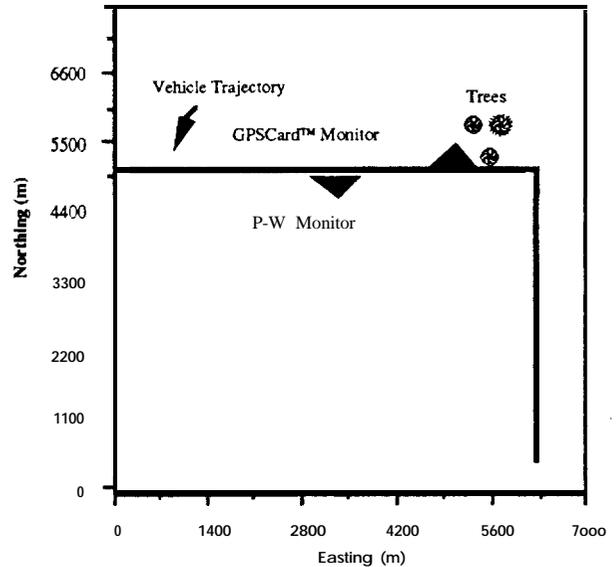


Figure 1: Vehicle Trajectory - Springbank Baseline

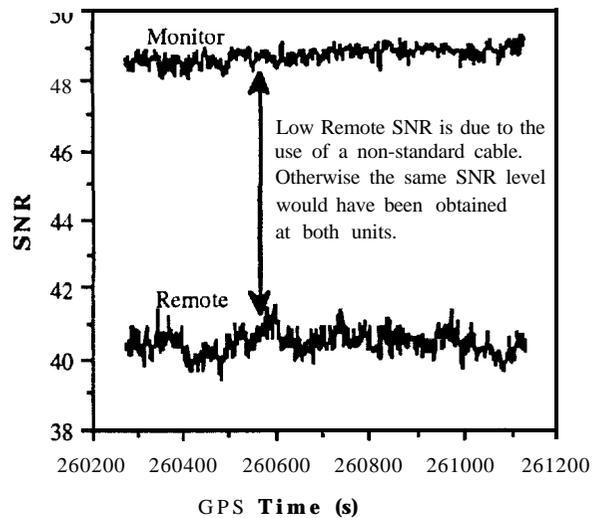


Figure 2: SRN of SV28 during Test. The Section of the Trajectory affected by Trees was observed between 260500s and 260700s.

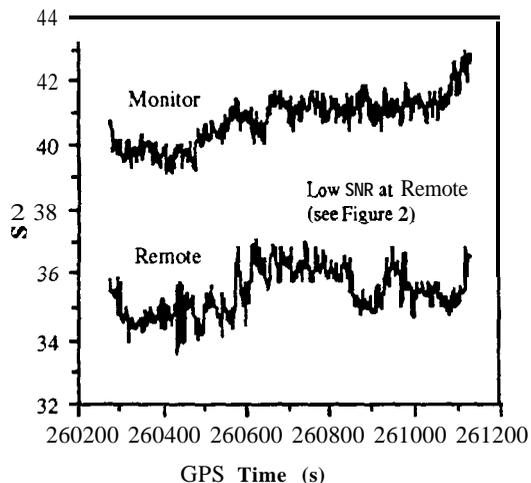


Figure 3: SRN of SV11 during Test. The Section of the Trajectory affected by Trees was observed between 260500s and 260700s.

Table 2 summarizes the ambiguity on the fly resolution results with both receiver types for various segments of the trials and under different a priori assumptions. In order to obtain numerous trials for testing the ambiguity resolution on the fly using the kinematic portion of the trajectory, the starting point of each trial was shifted 10 second forward from the previous trial. The 10 second shift is sufficient to decorrelate code and carrier receiver noise and multipath on the vehicle. Multipath effects at the monitor will not however be completely decor-related over such a short period. Up to 56 trials could be obtained for each case, depending upon the number of epochs required to resolve the ambiguities.

For each trial, an initial differential code solution was estimated to define the integer ambiguity range for each double difference pair. This search cube, which must contain the correct solution, is defined by the differential code solution and its estimated standard deviation, e.g., $\text{latitude}_{\text{diff}} \text{ code} \pm 3\sigma_{\text{lat}}$ for a 99% probability level. In the present case, the cube size varied from approximately eight m^3 for the P-XII P code and the GPSCard™ C/A code solutions to 1000 m^3 for the P-XII C/A code solution. An 8 m^3 cube typically generates 5000 integer combinations from which the correct one has to be isolated. The size of the initial cube is therefore important to decrease the computations required. This is why the computation time ratio shown in Table 2 for the P-XII C/A code case is substantially higher than any other case involving the GPSCard™ or the P-XII P code data.

The results for the clear and tree portion of the trajectory are given separately in Table 2 to explicitly show the performance degradation due to carrier phase multipath caused by the trees. The multipath causes the carrier phase residuals to increase, as shown in Figure 4, 5 and 6. In order for the statistical tests described earlier to select the correct solution with an acceptable reliability level, data cumulated over a longer period of time is required.

Various a priori standard deviations for the double difference carrier phase observations were tested. In the case of the GPSCard™, the use of a standard deviation of 10 mm for the $\nabla\Delta\phi$'s results in a 100% success rate for both portions of the trajectory. When a standard deviation of 5 mm is used, the observation time needed is reduced by a factor of six to seven, e.g., 15 versus 106 seconds for the clear portion of the trajectory. However, the level of success drops from 100% to 80% in this case. In the case of the tree portion of the trajectory, the success rate drops to an unacceptable level of 43%. A success rate less than 100% means that either no solution was found or a wrong solution was identified. This typically occurs when the non-gaussian effect of multipath biases the variance ratio tests. Single frequency L1 data is especially susceptible to this effect due to the relatively short wavelength (19 cm). The ambiguity resolution observation time needed for each successive trial separated by 10 seconds is shown in Figure 7. The time required is at the 100 seconds level during the clear portion of the trajectory and jumps to 600 seconds when the tree portion of the trajectory begins.

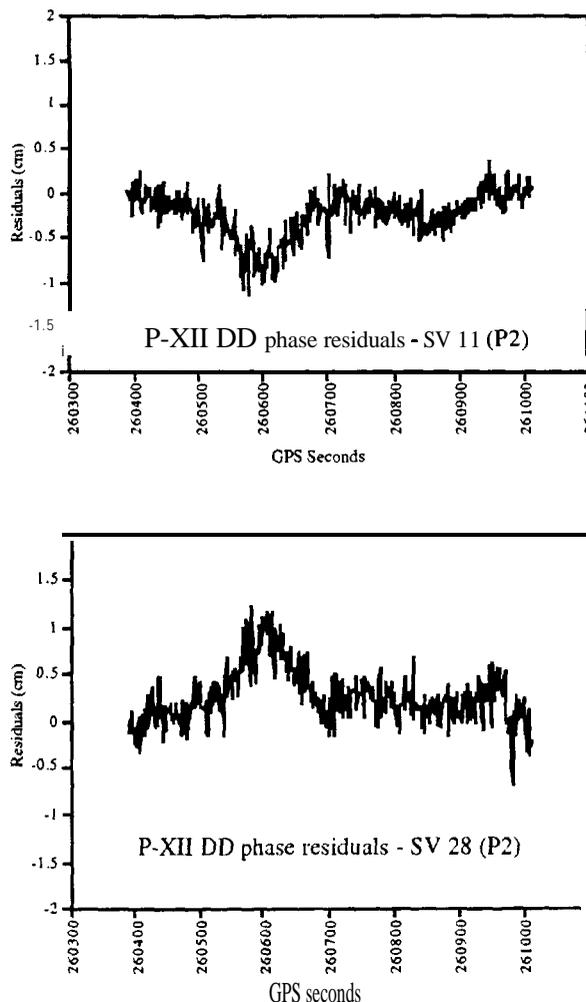


Figure 4: P-XII Double Difference Carrier Phase Residuals on L2 for SV 11 and 28

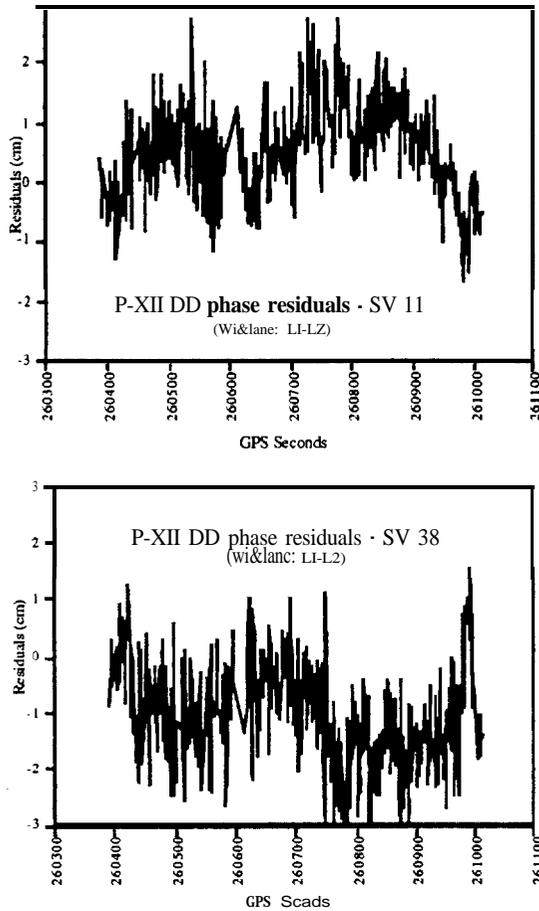


Figure 5: P-XII Double Difference Carrier Phase Residuals on (LI-L2) for SV 11 and 28

The performance of the P-XII C/A code trials are similar to that of the GPSCard™ in term of time to resolution. The computation time is however 20 times higher due to the much larger initial search cube as discussed earlier. The overall performance of the P-XII P1 and P2 results are about the same as that of the GPSCard™, in terms of time to resolution and success rate. The ambiguity resolution observation time needed for each successive trial separated by 10 seconds is shown in Figure 8 for the P-XII P1 solutions. Although many trials were unsuccessful in this case, the effect of the trees is similar to that seen for the GPSCard™ in Figure 7.

Widelaning of the carrier phase observations gives the best results. The ambiguities are resolved using one to three seconds of observations and the computation time is only 1% of that required for the first trial. This is due to the 86 cm wavelength of the widelane carrier which reduces the number of possible solutions and, therefore, the computation time (e.g., Lachapelle et al 1991). Also, widelane observations are relatively less sensitive to noise and multipath effects. The ambiguity resolution observation time needed for each successive trial separated by 10 seconds is shown in Figure 9 for the widelane case. The presence of multipath caused by the nearby trees can no longer be detected and the success rate during that

portion of the test is 92%. Since the observation time is much shorter when using the widelaning technique, several quasi-independent solutions can be obtained for the ambiguity set over relatively short time intervals, substantially increasing the reliability of the ambiguity set.

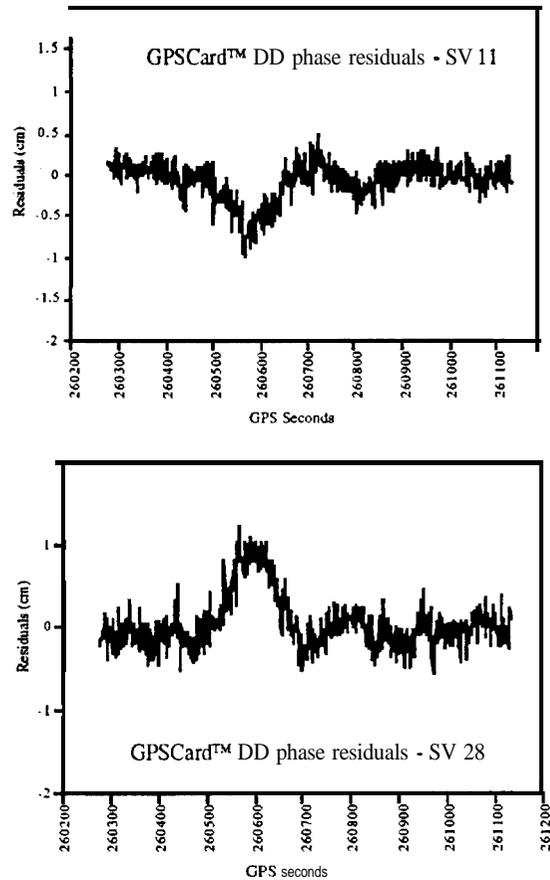


Figure 6: GPSCard™ Double Difference Carrier Phase Residuals for SV 11 and 28

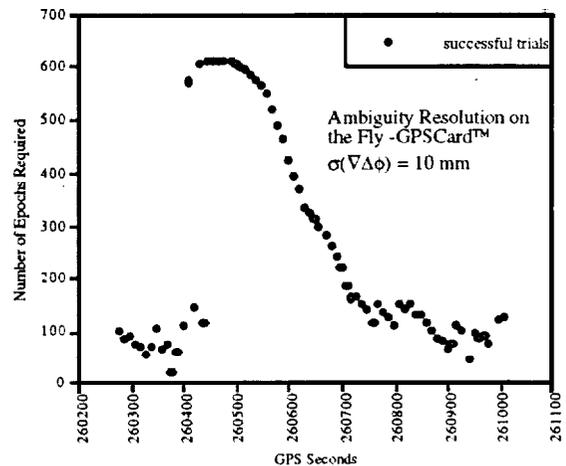


Figure 7: Ambiguity Resolution on the Fly Using GPSCard™ Data

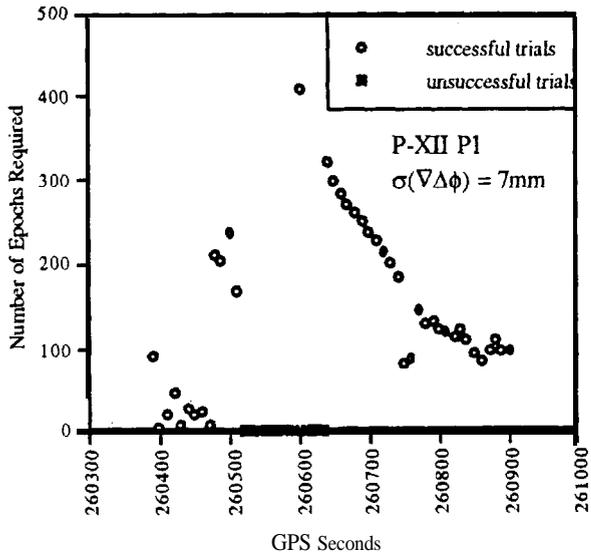


Figure 8: Ambiguity Resolution on the Fly Using P-XII P1 Data

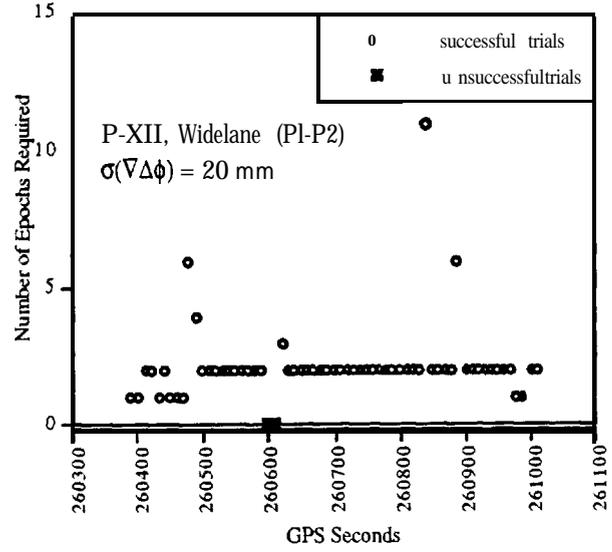


Figure 9: Ambiguity Resolution on the Fly Using P-XII Widelane (P1-P2) Data

TABLE 2:
AMBIGUITY RESOLUTION ON THE FLY - PERFORMANCE STATISTICS

Observation Area Type	$\sigma_{\Delta\nabla\phi}$	Nbr of trials	Nbr of Correct Trials	Success rate	Average Epochs Required	Computation Time ratio*
GPSCard™						
trees	10 mm	30	30	100%	500	1.00
clear	10 mm	43	43	100%	106	0.33
trees	5mm	30	13	43%	83	0.22
clear	5mm	56	45	80%	15	0.08
P-XII						
C/A and Φ	clear	5mm	6	100%	62	15.10
P1 and Φ	trees	10 mm	26	62%	282	0.89
	clear	10 mm	17	100%	167	0.39
	trees	7mm	26	58%	253	0.60
	clear	7mm	26	100%	85	0.30
P2 and Φ	trees	10mm	26	69%	157	0.37
	clear	10 mm	37	95%	28	0.08
P2 and wide lane (L1-L2)	trees	20 mm	26	92%	2.3	0.01
	clear	20 mm	37	100%	2.1	0.01

* Normalized with respect to first trial.

CARRIER PHASE SMOOTHING OF THE CODE SOLUTION

An interesting question is what would the level of performance be if instead of attempting to resolve the carrier phase ambiguities to their correct integer values, a carrier phase smoothing of the code solution were used. Such a solution is less accurate than a solution with the ambiguities resolved but much more robust and therefore better suited to many operational situations (e.g., Cannon et al 1992a). This question is particularly relevant when using single frequency data since the results described above indicate that the reliability of ambiguity resolution on the fly solutions obtained with such data is likely too low for operational use. Results obtained with the GPSCard™ units are shown in Figure 10. These were obtained with C³NAV, a software package developed at The University of Calgary (Cannon et al 1992a). The rms differences are 12 cm and 33 cm for the latitude and longitude, respectively. The corresponding rms difference for the height component is 21 cm. These results show that such a robust solution can yield an accuracy level at the 30-50 cm level in land kinematic mode using high performance CIA code receivers.

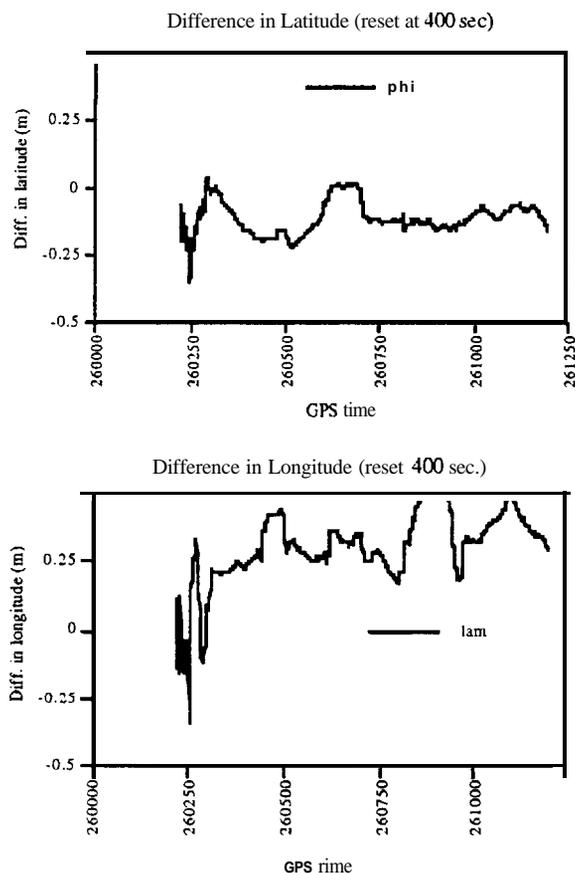


Figure 10: Carrier Phase Smoothing of the Code Solution

CONCLUSIONS

From the test results presented herein, one can conclude that both P code and high performance single frequency C/A code receiver technologies have the same level of performance in terms of ambiguity resolution on the fly when single frequency P code data is used. Both receiver technologies appear to be equally affected by carrier phase multipath. Under benign multipath conditions, the carrier phase ambiguities can generally be resolved in less than 100 seconds in land kinematic mode. Ambiguity resolution time and success rate is however sensitive to the standard deviation assigned to the carrier phase measurements. An optimistic standard deviation reduces substantially the ambiguity resolution time but, at the same time, decreases the success rate. As a consequence, the reliability of single frequency data to resolve ambiguities on the fly is relatively low, especially in an operational environment.

The above shortcomings can- however be dealt with effectively using the full capability of the dual frequency P code through widelaning. Although the a priori variance of the widelane carrier phase measurements is relatively high, the ambiguities can be generally be resolved successfully within a few seconds of observation. Although the P code is not expected to be available from 1994 onwards, widelaning will still be possible using cross-correlation and other patented techniques, such as the P-W technique being implemented by Ashtech (Ashtech 1992), to recover the full wavelength carrier phase on L2, and the code or a combination thereof on L1/L2. The latter method appears especially promising for kinematic applications in view of its relatively higher SNR as compared to the other techniques.

These findings reported in this paper are based on land kinematic trials and should not be automatically extrapolated to other kinematic modes, due primarily to different multipath behavior. For example, recent experiments with the same receivers as used herein on board a survey launch lead to significantly different results. The time to resolution was higher and the success rate lower due to the strong reflective properties of sea water (Lachapelle et al 1993).

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