

HIGH PRECISION C/A CODE TECHNOLOGY FOR RAPID STATIC DGPS SURVEYS

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ABSTRACT

The use of code-only and carrier phase measurements for rapid static surveys is investigated using the NovAtel **GPSCard™**, a new receiver technology which measures C/A code with an accuracy of 10 cm and uses a narrow correlator spacing on the code tracking loops to reduce code multipath effects. The use of double differenced code measurements deliver a positioning accuracy of some 10 to 20 cm (rms) in each of the three coordinates using a few minutes of data. In the case of short baselines, for which the integer values of the carrier phase ambiguities can be recovered, the above code solutions provide a relatively small search volume which reduces the integer carrier phase ambiguity search time to several seconds on a laptop PC, using a least-squares search technique. Results presented for baselines of 700 metres and 4 kilometres show that the correct ambiguities can be recovered using some two and five minutes of data, respectively.

1. INTRODUCTION

The use of a few minutes of double differenced C/A code observations obtained with standard geodetic receivers typically yield a positioning accuracy of several metres, even for short baselines, and is limited by the 1-2 m noise level of each receiver and by prevailing code multipath. The carrier phase data can yield cm-level accuracy relative positions after a few minutes of observations provided that double

differenced carrier phase ambiguities can be resolved. Several methods are available to solve the carrier phase ambiguities using observations made over periods as short as a few minutes. The most well known are the ambiguity function method (e.g., Mader 1990) and several least-squares search methods (e.g., Frei & Beutler 1990, Hatch 1990, Remondi 1991). The computational effectiveness of these methods is a function of the quality of the initial approximate solution which defines the size of the search area and therefore, the number of trial solutions. The approximate solution is obtained through a pure code or carrier phase smoothed code solution. The accuracy of these code solutions has been limited up to now by the 1-2 m C/A code measuring noise of current receivers and by the effect of field multipath. Under the current constraints, the correct ambiguity search requires up to several minutes of computation time on a 386 microprocessor-based PC.

A new C/A code receiver technology, unveiled in September 1991 (Fenton et al 1991) by NovAtel Communications Ltd., Calgary, has the potential to improve both the accuracy of code only solutions and the effectiveness of ambiguity search methods by reducing the size of the search volume. The two major relevant characteristics implemented in the 10-channel NovAtel **GPSCard™** are a 10-cm C/A code measuring accuracy and a narrow correlator spacing on the code tracking loops to increase resistance to code multipath (Van Dierendonck et al 1992). The objective of this paper is to test the performance of this new technology under normal operating

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field conditions for rapid differential static surveys using code only and carrier phase measurements. In the latter case, the code solutions are used to reduce the search volume in order to find the correct integer ambiguities in a relatively short period of time.

2. C/A CODE NOISE AND FIELD MULTIPATH EFFECTS

The C/A code measuring noise of the GPSCardTM was previously analysed using a single antenna/two receiver test to isolate receiver measuring accuracy from multipath (Erickson et al 1991). Figure 1, which is representative of the results obtained, shows that after a brief convergence period, the receiver noise indeed settles in a range below the 10 cm level.

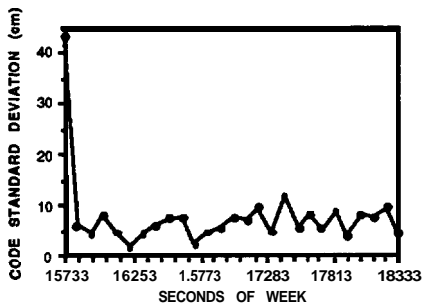


Figure 1: Instantaneous GPSCardTM C/A Code (RMS) Noise

The effect of field multipath on C/A code measurements was analysed by Cannon & Lachapelle (1992) using a semi-kinematic DGPS test. The use of choke ring ground planes to reduce multipath was investigated. Double differenced code and carrier measurements were intercompared to derive the combined effect of measuring code noise and multipath. The combined rms effect on a single code measurement when a choke ring ground plane was used at the monitor station only was 29 cm. The corresponding value when choke ring ground planes were used at both the monitor and the remote stations was 22 cm. If the above 10 cm receiver measuring noise is removed, the remaining rms multipath effect is 27 and 20 cm, respectively. These values are relatively low and indicate that a positioning accuracy of the order of a few decimetres should be achievable in DGPS static mode for short baselines using a few minutes of code data.

3. AMBIGUITY SEARCH METHODS

The carrier phase ambiguity search concept is shown in Figure 2. The coordinates at the monitor station are held fixed. An approximate code or carrier phase smoothed code solution provides the centre of the search area. The size of the search cube is a function of the accuracy of the approximate solution. Since the search area must contain the correct but yet unknown solution corresponding to the correct ambiguity set, the sides of the search area are typically set at three times the estimated standard deviations of the approximate coordinates. The larger the initial search cube, the higher the number of potentially correct integer ambiguity solutions to calculate and test, and the longer the computation time required. The use of C/A code solutions with standard geodetic receivers may require up to several minutes of computation on a 386 microprocessor-based PC.

Two ambiguity search methods were initially considered herein, namely the ambiguity function method (AFM) and a least-squares search method. Since both were found to be fundamentally equivalent, as it will be shown below, a least-squares search method was implemented.

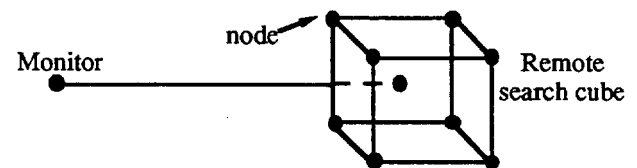


Figure 2: Carrier phase ambiguity search concept

Least-squares Ambiguity Search

In this method, the property that the quadratic form of the carrier phase residuals is minimum at the correct solution is used. This minimum condition is realized provided the residuals are normally distributed. Four primary satellites with a relatively strong geometry are used to compute sets of double difference ambiguities $\{AVN = \Delta\nabla\rho - \Delta\nabla\Phi\}$ at each one of the eight nodes of the search area and to determine the range of integer ambiguities to take into account. For each ambiguity combination, the coordinates of the remote station are calculated using the

primary satellites and used in turn to calculate the ambiguities of the secondary satellites. In the approach implemented for this paper, the solution with the smallest quadratic form (vTv) of the carrier phase residuals is deemed to be the correct one, provided the quadratic form is at least two times smaller than the next smallest quadratic form. The nearest integer values of the ambiguities are then held fixed to calculate the final double difference carrier phase solution. Other statistical criterias than the one selected above can be used (e.g., Frei & Beutler 1990).

Ambiguity Function Method

The ambiguity function, commonly used in radar applications, was first introduced in GPS data processing by Counselman & Gourevitch (1981). The ambiguity function, for the single frequency case, can be written as

$$A(x,y,z) = \sum_{k=1}^K \sum_{j=1}^{M-1} e^{i\theta}$$

where

$$\theta = 2\pi[\Delta\nabla\phi^{jl}(x,y,z)_{obs} - \Delta\nabla\phi^{jl}(x_0,y_0,z_0)_{calc}]$$

$\Delta\nabla\phi^{jl}(x_0,y_0,z_0)_{obs}$ is the observed $\Delta\nabla\phi$ in cycles on satellite j (with respect to the base satellite, 1) at the correct but unknown position (x,y,z) , $\Delta\nabla\phi^{jl}(x,y,z)_{calc}$ is the corresponding double difference calculated at the approximate position (x_0,y_0,z_0) , so θ is the double differenced phase residual in radians. M denotes the number of satellites tracked and K, the number of observation epochs. The ambiguity function, $e^{i\theta}$, is equivalent to $(\cos\theta + i \sin\theta)$, and since we are only interested in the real component, the ambiguity function can be simplified to

$$A(x,y,z) = \sum_{k=1}^K \sum_{j=1}^{M-1} \cos\theta$$

The function is invariant under integer cycle changes, i.e., $e^{i(\theta + 2\pi n)} = e^{i\theta}$, hence cycle slips

do not reduce its effectiveness. The theoretical maximum magnitude of A is $K(M-1)$, however this maximum would only be reached if all residuals are zero, therefore the practical maximum is shown as

$$A_{max} \approx K(M-1) \text{ where } [x_0,y_0,z_0] = [x,y,z]$$

If sufficient measurements (K's and/or M's) are available, a maximum will be obtained at one point only, namely at the correct position $[x_0,y_0,z_0]$. To find A_{max} , values of A are calculated for each of the integer ambiguity combinations defined from the approximate search cube.

Equivalence of Ambiguity Function and Least-Squares Search Methods

When the ambiguity function method is used, the sum of the cosine of the residuals, i.e., $\sum \sum \cos\theta$, is maximized. The use of the least-squares search minimizes the quadratic form of the residuals, i.e., $\sum \sum v^2$. The cosine function can be expanded as a Taylor series to give

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

Neglecting the higher order terms yields the ambiguity function as

$$\sum_{k=1}^K \sum_{j=1}^{M-1} \cos\theta \approx \{ K(M-1) - \frac{1}{2} \sum_{k=1}^K \sum_{j=1}^{M-1} \theta^2 \}$$

The sum of the cosine of the residuals is evidently maximized when the quadratic form of the residuals itself is minimized, which is the same condition as in the least-squares search technique. The **least-squares search**'s sum of the squares of the residuals can therefore be derived from the ambiguity function (in **metres²**):

$$\sum_{k=1}^K \sum_{j=1}^{M-1} v^2 \approx 2 \{ K(M-1) - A_{max} \} \frac{\lambda^2}{(2\pi)}$$

4. RESULTS AND ANALYSIS

Two short baselines observed on February 12 and 17, 1992, respectively, were tested. The two baselines, observed in the Calgary area, were 700 m and 4 km in length, respectively. Six satellites with an elevation $> 10^\circ$ were available during the 20 to 30 minute observation periods selected. Choke ring ground planes were used at both stations in both cases. The PDOP varied between 2.8 and 3.5 during these periods. The *ground truth* was provided in each case by a standard carrier phase double difference solution using the entire observation period. The University of Calgary's SEMIKIN software package (Cannon 1990) was used to obtain these solutions which are estimated to be accurate within a few cm. The double difference code-only and carrier phase solutions presented below were also obtained with a newly modified version of SEMIKIN. The least-squares ambiguity search described in the previous section was implemented in the software.

The two- and five-minute code solutions for the 700-m and 4-km baselines are shown in Figures 3 and 4, respectively. In the first case, the 20-minute observation period was divided into ten two-minute segments. In the second case, the 25-minute observation period was divided into five five-minute segments. Ten two-minute and five five-minute solutions were obtained for each case, respectively. In each case, the coordinate errors reach some 30 cm. The overall rms for any one coordinate are 17 and 12 cm for the 700-m and 4-km baselines, respectively.

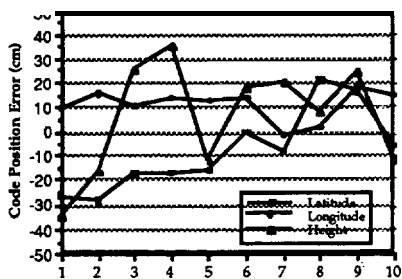


Figure 3: Two-minute code solutions for the 700-m baseline. Six satellites were available at an elevation $> 10^\circ$ and with a PDOP between 2.8 and 3.5.

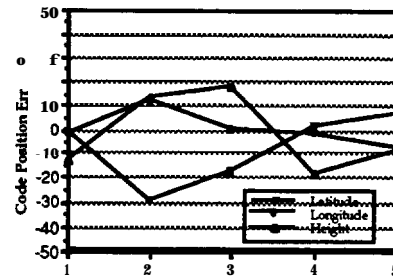


Figure 4: Five-minute code solutions for the 4-km baseline. Six satellites were available at an elevation $> 10^\circ$ and with a PDOP between 2.8 and 3.4.

The corresponding carrier phase solutions are presented in Tables 1 and 2, respectively. The code solutions obtained previously were used to define the initial search cube. In most cases, this area was of the order of 30 cm x 30 cm x 30 cm. This resulted in a number of potential solutions less than 30 in most cases. The computation time required for a 386 microprocessor-based, 20 MHz clock rate, PC to search for the correct set of ambiguities was of the order of 12 to 15 seconds. The corresponding time on a 486 microprocessor-based, 33 MHz clock rate, PC was five to seven seconds. The use of a relatively smaller initial search area thus reduced the computation time substantially. In the case of the 700-m baseline, the correct ambiguities were found in each one of the 10 trials. In the case of trial 1 and 8, however, the ratio between $\mathbf{v}^T\mathbf{v}$ for the second best solution and $\mathbf{v}^T\mathbf{v}$ for the best solution was less than 2 and, therefore, statistically less significant. The fact that the correct solution was selected is more of a matter of chance in these two cases. This also shows that, under the prevailing multipath conditions and satellite geometry, a 2-minute observation span is a lower limit which would have to be increased by a few minutes to provide sufficient reliability under operational conditions.

In the case of the 4-km baseline, the correct ambiguities were found in four of the five trials. However, the ratio between $\mathbf{v}^T\mathbf{v}$ for the second best solution and $\mathbf{v}^T\mathbf{v}$ for the best solution was less than 2 for three of the five trials, indicating that the observation period should be increased by a few minutes to increase the reliability.

Table 1: Two-minute carrier phase solutions for the 700-m baseline. **The ambiguities were found using a least-squares search technique.**

Trial	$\Delta\phi$	Ah	Ah
*1	0.3 cm	0.0 cm	0.0 cm
2	0.6	-0.2	0.6
3	0.3	-0.2	0.5
4	-0.3	0.0	-0.4
5	-0.3	0.0	0.1
6	0.0	0.0	0.3
7	0.3	0.0	0.5
*8	0.3	0.0	0.3
9	-0.3	0.4	-0.8
10	0.0	0.2	-0.2

* $\sqrt{\mathbf{v}^T \mathbf{v}}_{\text{second best solution}} / \sqrt{\mathbf{v}^T \mathbf{v}}_{\text{best solution}} < 2$

Table 2: Five-minute carrier phase solutions for the 4-km baseline. **The ambiguities were found using a least-squares search technique.**

Trial	$\Delta\phi$	$\Delta\lambda$	Ah
*1	1.5 cm	-0.8 cm	0.1 cm
2	0.9	-0.4	-1.4
3	0.6	-0.8	-1.9
*4	2.5	14.4	23.7
*5	0.6	-1.2	-1.1

* $\sqrt{\mathbf{v}^T \mathbf{v}}_{\text{second best solution}} / \sqrt{\mathbf{v}^T \mathbf{v}}_{\text{best solution}} < 2$

5. CONCLUSIONS

The high C/A code measuring accuracy and high code multipath rejection capability tested herein can provide ambiguity-free and operationally robust code solutions accurate to a few decimetres for short baselines. In the case of longer baselines, the error contribution of the code noise and multipath will remain the same but atmospheric and orbital errors will result in

an error component which grows as a function of the baseline length. The use of carrier phase-smoothed code data (e.g., Lachapelle et al 1987) to improve further this type of solution is currently being investigated.

In the case of short baselines for which the integer values of the carrier phase ambiguities can be recovered, the code-only solutions provide a relatively small search cube which reduces the ambiguity search time to several seconds on a laptop PC. The results shown here indicate that the correct ambiguities can be recovered using observation periods of a few minutes. The availability of a more complete constellation will improve reliability and effectiveness. The use of the new single frequency C/A code technology tested herein for rapid static surveys shows much potential. Carrier phase ambiguity resolution without static initialization should also likely be possible using an adequate satellite geometry and is also currently being investigated.

6.0 ACKNOWLEDGMENT

Part of this research was conducted with the financial assistance of the Geodetic Survey Division, Canada Centre for Surveying, Energy Mines and Resources Canada, Ottawa.

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