Performance Analusis of A Shipborne Gyrocompass With A Multi-Antenna GPS System

G. Lu, G. Lachapelle, M. E. Cannon
Department of Geomatics Engineering,
The University of Calgary, Alberta

B. ***Vogel,*** Defence Research Establishment,
Department of National Defence, Victoria, B.C.

ABSTRACT

A four-receiver configuration was used to estimate the three attitude parameters and assess the headings obtained with a Sperry Mark 3 Model C gyrocompass mounted on a 1500 tonnes, 72-m vessel. The GPS configuration consisted of four lo-channel NovAtel GPSCard™s operating in independent mode. The antenna separations varied between 5 and 15 m. The antenna locations in the ship coordinate system were obtained through a combination of a conventional and GPS survey. The remaining misalignment between the gyrocompass and the multi-receiver configuration was estimated by comparing GPS and gyrocompass heading measurements made throughout the sea trial. The formulation used to estimate the above attitude parameters is described. The multi-receiver data was processed with The University of Calgary's MULTINAV™ attitude software. The performance of the gyrocompass was assessed using 10 hours of data collected over a two-day period in Spring 1993 under a wide range of ship manoeuvres such as acceleration, deceleration and 360" turns. The estimated double difference carrier phase residuals, which provide a measure of receiver carrier phase noise and multipath, were below the 5 mm level. The estimated accuracy of the GPS-derived heading parameter was 2.2 arcmins or 0.04" and the standard deviation of the differences between GPS and gyrocompass headings during straight trajectory segments was 0.16". The use of a two-receiver configuration to determine the heading component was shown to deliver a level of accuracy similar to that obtained with the four-receiver configuration.

INTRODUCTION

The major objective of this paper is to assess the performance of a multi-antenna GPS system to measure the ship's attitude parameters, namely the heading, pitch and roll parameters, with emphasis on the heading component. The ship selected for the experiment was the 1,500 tonnes, 72-metre Endeavour, of the Canadian Department of National Defence. The GPSderived heading measurements were compared to those obtained with the Sperry Mark 23 Model C gyrocompass installed on the ship, under various ship's dynamics. In order to obtain sufficient redundancy, a four-antenna GPS system was used. This was important since no external information was available to assess the GPS-derived roll and pitch components. In order to assess the minimal GPS configuration required for heading determination, the heading measurements obtained from a two-antenna system were compared to those obtained with the full four-antenna system to assess the relative accuracy degradation, if any, of the two-antenna system.

The GPS sensor type selected for the experiment was the NovAtel GPSCard™, equipped with chokering groundplanes to reduce multipath effects. The GPSCard™ is a single frequency, C/A code, 10-channel unit which uses a patented narrow correlator spacing technique to reduce C/A code receiver noise and multipath (e.g., Van Dierendonck et al 1992, Cannon & Lachapelle 1992). The use of chokering groundplanes further reduces multipath, which is relatively high in the marine environment due to the high conductivity and high dielectic constant of sea water (Lachapelle et al 1993). The carrier phase tracking loops of the GPSCard™ have been shown to be very stable in the marine environment (Lu et al 1993), an important characteristic for shipborne attitude determination.

GPS ANTENNA SETUP AND FIELD MEASUREMENTS

The four GPS antennas with their chokering groundplanes were deployed around the helicopter deck of the Endeavour as shown in Figure 1. Three of the four antennas were mounted on the top ends of long thin supporting steel rods while the remaining antenna (Ant 2) was mounted on a solid metal plate. Even though the rods were fastened to the deck using wires, remaining vibration effects may exist due to strong winds and to the motion of the ship.

The antenna positions in the ship coordinate system were obtained through a conventional survey while the ship was tied to the wharf. A theodolite was set up on the ship and centered on the ship's centre line. The y-axis of the ship coordinate system is defined by the ship's centre line in the horizontal plane, pointing aft of the ship. This direction defies the heading of the ship. The x-axis lies in the horizontal plane and points to starboard. The z-axis then forms a right-handed system with the x and y axes. The origin is at Antenna 1. Since the ship was moving due to the wind and ocean tide, it was difficult to take accurate vertical angle readings, hence the surveyed antenna coordinates are accurate to 1 to 2 cm. GPS-derived distances were also computed by averaging the distances obtained by an on-the-fly carrier phase resolution technique using 90 minutes of data. Table 1 shows both the surveyed and GPS-derived distances between the antennas. Using the GPS-derived distances between the four antennas as weighted constraints, a least-squares adjustment of the conventional survey measurements was performed to obtain the coordinates of the antennas in the ship's coordinate system.

The sea measurements were conducted on May 31 and June 1. Six hours of data collected on May 31 and four hours of data collected on June 1,1993, during which various manoeuvres were performed by the ship were selected for post-processing and analysis. The carrier phase data was collected every 1 s on May 31 and every 0.5 s on June 1. The gyrocompass measurements were time tagged with GPS time and collected every 2 seconds.

The GPS data post-processing was done using MULTINAV™, a program developed at The University of Calgary (Lu et al 1993). In MULTINAV™, the distances between the antennas are held fixed to determine the carrier phase ambiguities on the fly and the three attitude parameters are determined independently at each carrier phase measurement epoch. The carrier phase tracking bandwidth on each GPSCard™ was set at 15 Hz. Since carrier phase measurements are not affected by damping from platform dynamics, the GPS-derived attitude parameters obtained with MULTINAV™ will not be damped by ship's dynamics.

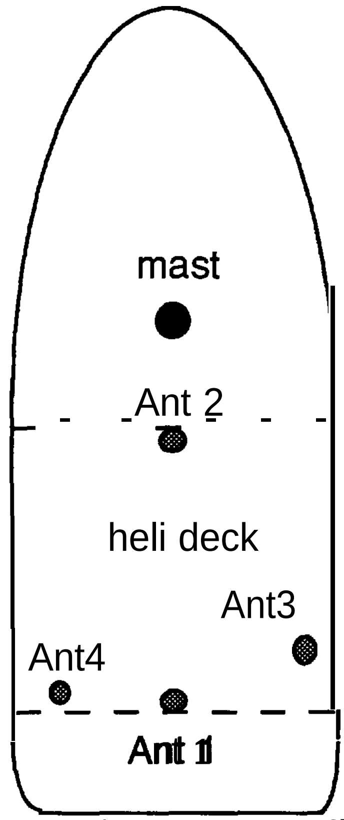

Figure 1: Antenna Layout on Ship

Table 1: Distances between GPS Antennas

Baseline	Surveyed value	GPS-derived	Difference
1-2	14.858m	14.865m	-0.007m
1-3	5.289	5.291	-0.002
1-4	4.830	4.842	-0.012
2-3	13.841	13.840	0.001
2-4	15.471	15.473	-0.002
3-4	9.660	9.672	-0.012

ATTITUDE ESTIMATION USING GPS

Attitude is defined as the orientation of the specified body frame with respect to a reference frame. In our case, the body frame is defined as the ship coordinate system and the reference frame is defined as the local level coordinate system, i.e., north (Y-axis), east (X-axis) and up (Z-axis). Through three consecutive rotations, the local level coordinate system can be transformed into the ship coordinate system. The rotation matrix is expressed as (Wertz 1978)

$$\begin{pmatrix} x^b \\ y^b \\ z^b \end{pmatrix} = R_2(\phi)R_1(\theta)R_3(\psi)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

$$R_2(\phi)R_1(\theta)R_3(\psi) = R(\psi,\theta,\phi) = \tag{2}$$

$$\begin{pmatrix} c(\phi)c(\psi)-s(\phi)s(\theta)s(\psi) & c(\phi)s(\psi)+s(\phi)s(\theta)c(\psi) & -s(\phi)c(\theta) \\ -c(\theta)s(\psi) & c(\theta)c(\psi) & s(\theta) \\ s(\phi)c(\psi)+c(\phi)s(\theta)s(\psi) & s(\phi)s(\psi)-c(\phi)s(\theta)c(\psi) & c(\phi)c(\theta) \end{pmatrix}$$

where $R(\psi,\theta,\phi)$ is an orthogonal matrix which rotates the GPS-derived local-level coordinates $(x, y, z)^T$ of an antenna to its corresponding body frame coordinates $(x^b, y^b, z^b)^T$. The functions s() and c() are sine and cosine functions, respectively. The three rotation angles (ψ,θ,ϕ) are called heading, pitch and roll. In GPS attitude determination, we know the GPS-derived local-level coordinates $(x, y, z)^T$ of all the antennas and we want to find the heading, pitch and roll of the antenna platform defined by the specified three or more GPS antennas.

In order to specify a platform or plane in space, two non-collinear baseline vectors are needed, which in turn requires at least three GPS antennas located in a proper configuration. Assume that a rigid antenna platform is defined by Ant 1, Ant 2 and Ant 3, as shown in Figure 1, and that a coordinate system is set up with its origin at Ant 1, the y-axis running along the baseline from Ant 1 to Ant 2, the x-axis pointing to the right of y-axis and lying on the antenna platform, and the z-axis then forming a right hand system with x and y axes. This coordinate system is called the antenna platform coordinate system. The attitude of this antenna platform with respect to the local level coordinate system can then be computed using the direct computation method or the least-square estimations method (Lu et al 1993). The direct computation formulas are

$$\psi = -\tan^{-1}(x_2 / y_2) \tag{3}$$

$$\theta = \tan^{-1}(z_2 / \sqrt{x_2^2 + y_2^2}\,) \tag{4}$$

$$\phi = -\tan^{-1}(z_3'' / x_3'') \tag{5}$$

where (x_3'', y_3'', z_3'') are the coordinates of Ant 3 obtained after two rotations, the first rotation by a heading angle of ψ around the local level z axis, and the second rotation by a pitch angle θ around the rotated local level x-axis. The above direct computation method only uses the coordinates of the three antennas defining the platform and thus the estimates of the attitude parameters are sub optimal. This method does not require the antenna's body frame coordinates.

By differentiating equations (3), (4) and (5) with respect to x, y and z and applying the error propagation law to the resulting differentiated equations, the accuracy of the estimated attitude parameters can be approximated by

$$\sigma_\psi \le \sigma_{max}(x,y) / \sqrt{x_2^2 + y_2^2} = \sigma_{max}(x,y) / L_{12}\cos(\theta) \tag{6}$$

$$\sigma_\theta \le \sigma_{max}(x,y,z) / \sqrt{x_2^2 + y_2^2 + z_2^2} = \sigma_{max}(x,y,z) / L_{12} \tag{7}$$

$$\sigma_\phi \le \sigma_{max}(x_3'',z_3'') / \sqrt{x_3''^2 + z_3''^2} = \sigma_{max}(x_3'',z_3'') / L_{13}\cos(\alpha) \tag{8}$$

where $\sigma_{max}(x, y) = max(\sigma_x, \sigma_y)$, L_{12} is the Ant 1 - Ant 2 baseline length, L_{13} is the Ant 1 to Ant 3 baseline length, and *a* is the angle between baseline L_{13} and the x-axis. In the above derivations, the correlation between the coordinate components x, y, z are neglected.

Based on the multiple GPS antenna configuration on the ship and assuming a l-cm accuracy for the GPS-derived coordinate components x, y and z, the accuracy of the estimated attitude parameters by the direct computation method would be 2.4 arcmins for the heading, 2.3 arcmins for the pitch and 7.0 arcmins for the roll. These accuracy estimates are well within the anticipated accuracy of 10 arcmins required to assess the gyrocompass.

If the coordinates of the GPS antennas in the antenna platform coordinate system are precisely known through a survey or initialization process, a least-squares estimation for three rotation angles (ψ,θ,ϕ) can be employed. For each GPS antenna, we have the relationship

$$b_i = R(\psi,\theta,\phi)\, d_i \qquad i = 1,2,\ldots,n, \tag{9}$$

where $b_i = (x_i^b, y_i^b, z_i^b)^T$ is the known baseline vector for the i^{th} antenna in the antenna platform coordinate system and $d_i = (x_i, y_i, z_i)$ is the corresponding vector in the local level system derived from GPS measurements. Linearizing equation (9) with respect to $(\psi, 0, \phi)$, the optimal estimates (ψ,θ,ϕ) of the antenna platform can then be determined by a least-squares procedure using all the baseline vectors of the multiple GPS antennas. The approximate values of (ψ,θ,ϕ) used for the linearization process are obtained by direct computation using equations (3), (4) and (5).

The least-squares estimates of the attitude parameters are optimal since all the position information contained in the multiple GPS antennas is used. The accuracy of the estimated parameters are obtained through the diagonal elements of the inverse of the normal matrix of the least-squares procedure. For the configuration used in Figure ***1, the a posteriori*** standard deviations estimated from the least-squares estimation procedure were 2.2 arcmins for the heading, 2.3 arcmins for the pitch and 4.8 arcmins for the roll components.

Usually, the antenna platform coordinate system is not identical to the ship coordinate system whose attitude parameters are required. Therefore, the estimated attitude parameters (ψ,θ,ϕ) of the GPS antenna platform should be corrected for the discrepancies between these two systems. In this project, the antenna coordinates in the ship coordinate system were surveyed as described earlier. Using the survey-derived antenna coordinates, the rotation angles from the ship coordinate system to the antenna platform coordinate system can be computed. They are 1.025" for the heading (anti-clockwise positive), 13.875" for pitch (aft up positive), and 17.120" for the roll (left side up positive). In order to get the attitude of the ship coordinate system with respect to the local level system, these rotation angles are to be subtracted from the estimated attitude parameters of the antenna platform.

DATA ANALYSIS

May 31 Test

The data collected during the period from GPS time 171000 s to 192500 s was used. At least four satellites were tracked throughout the test. Usually, five or six satellites were available above a 5" elevation. All the data set could be therefore be processed successfully. A few cycle slips were detected on low elevation (< 12') satellites. The double difference carrier phase integer ambiguities were resolved on the fly using between a few and several seconds of data. Since no effort was made to optimize the configuration to accelerate ambiguity resolution, this is considered fully satisfactory.

Shown in Figure 2 is the horizontal trajectory of the ship. The maximum cruising speed reached about 13 knots during the manoeuvres. The GPS-derived heading measurements are shown in Figure 3. The heading differences between the GPS multi-antenna system and the ship gyrocompass are shown in Figure 4. The GPS-derived heading values, available every second, were linearly interpolated to obtain the obtain corresponding values at epochs where gyrocompass values were available.

After removing a mean difference of -1.86" between GPS-derived and gyrocompass headings, the standard deviation of the heading difference was calculated as 0.35'. The mean difference is caused by the misalignment between the two heading systems.

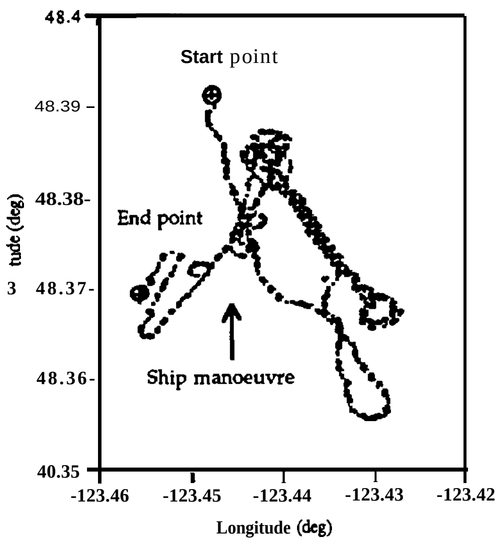

Figure 2: Ship Trajectory - May *31* Test

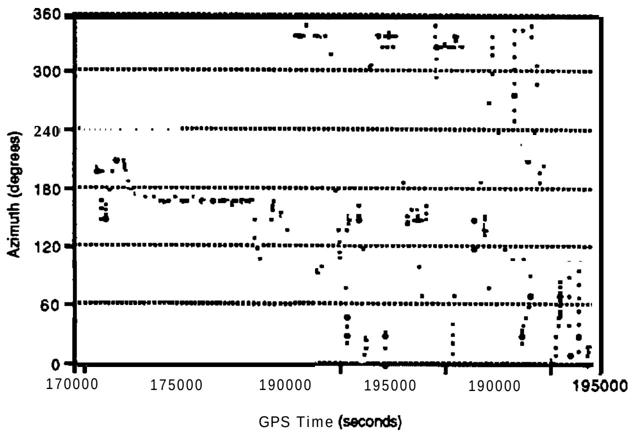

Figure 3: GPS-Derived Ship Heading - May 31 Test

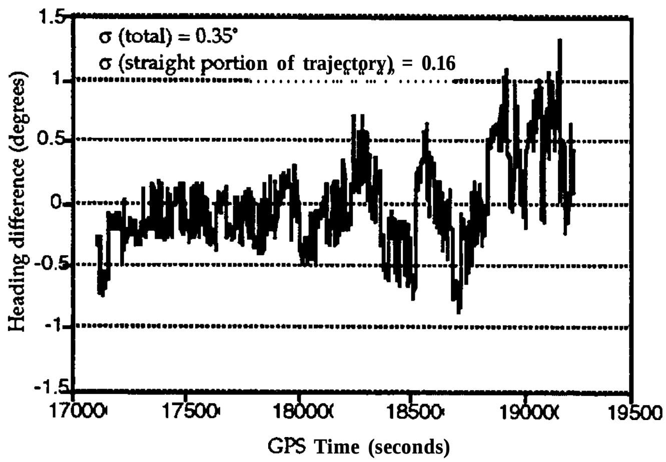

Figure 4: Heading Differences between GPS and Gyrocompass - May *31* **Test**

It can be seen from Figure 4 that the gyrocompass-derived headings are affected by damping caused by ship's dynamics. Figure 5 shows the ship heading change rate obtained by differentiating the GPS heading measurements. Comparing Figures 4 and 5, a strong correlation can be seen between gyrocompass damping and the heading change rate. As pointed out earlier, MULTINAV™ estimates the GPS-derived attitude parameters independently at each epoch and these are consequently free from platform dynamics. In order to obtain a representative measure of the agreement between GPS and gyrocompass-derived headings during a straight trajectory segment, the mean difference and its standard deviation were calculated for the period 172,900 s - 178,000 s. The values were found to be -1.93" and 0.16°, respectively. The latter value includes the GPS and gyrocompass errors. If one assumes that the GPS heading measurements were obtained with an estimated accuracy of 2.2 arcmins or 0.04°, as derived earlier, the accuracy of the gyrocompass heading during the straight trajectory portion selected herein, can then be estimated as 0.15".

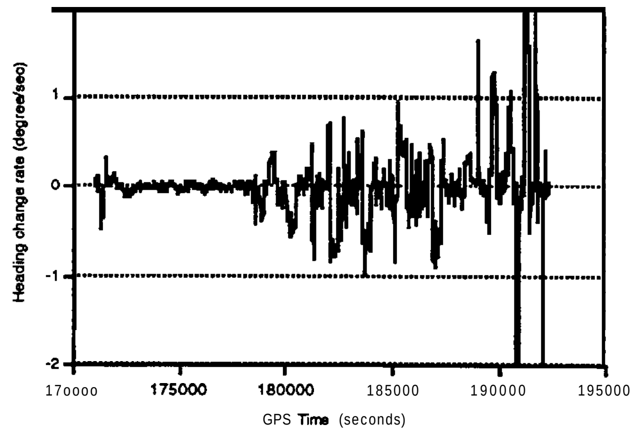

Figure 5: Heading Change Rate - May *31* Test

Shown in Figures 6 and 7 are the ship's pitch and roll components estimated with the GPS attitude system. A positive roll angle corresponds to a tilting of the ship towards starboard. A positive pitch angle corresponds to a tilting of the ship towards the stern. The 72-m ship was relatively stable in pitch and roll during the manoeuvres and the pitch and roll values are relatively small. The change of nearly 1" in roll occurring around 180,000 s is due to various factors, including a change of heading of 180", as can be seen in Figure 3.

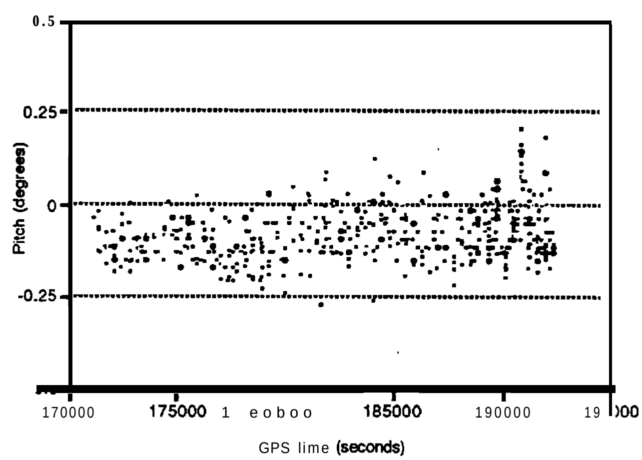

Figure 6: GPSDerive Pitch - May *31* Test

The rms double difference carrier phase ($\Delta\nabla\Phi$) residuals varied between 2 and 5 mm, which is well within the anticipated range for this type of receiver and antennas equipped with chokering groundplanes in the marine environment (e.g., Lachapelle et al 1993).

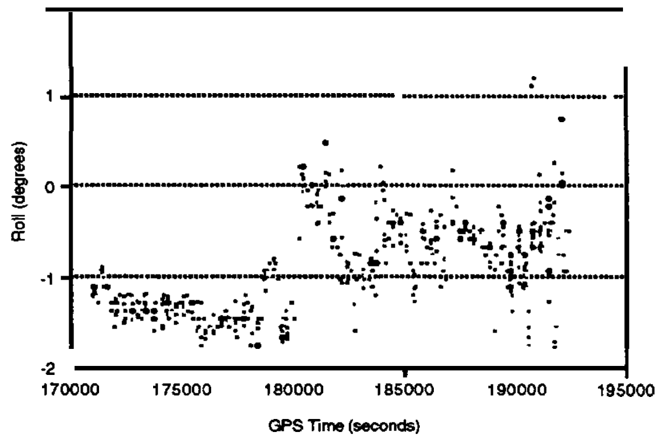

Figure 7: GPS-Derived Roll - May 31 Test

June 1 Test

The repeatability of the performance of the GPS multi-antenna attitude system were verified with the data collected on June 1. The GPS-derived ship heading measurements are shown in Figure 8. The differences between GPS and gyrocompass-derived headings are shown in Figure 9. The heading change rate is shown in Figure 10. Again, it can be seen from that the gyrocompass-derived headings are affected by damping due to ship's dynamics. Overall, the results have similar characteristics to those obtained on May 31.

Again, a representative measure of the agreement between GPS and gyrocompass-derived headings during a straight trajectory segment was obtained by calculating the mean difference and its standard deviation for the period 259,000 s - 267,000 s. The values were found to be -1.29' and 0.16°, respectively. The standard deviation of the difference is identical to that estimated for the previous day. The difference of 0.87" between the mean differences of the two days is within the estimated ship logging system initialization biases.

The pitch and roll components estimated for June 1 exhibits the same characteristics as those obtained on May 31 and are not shown here.

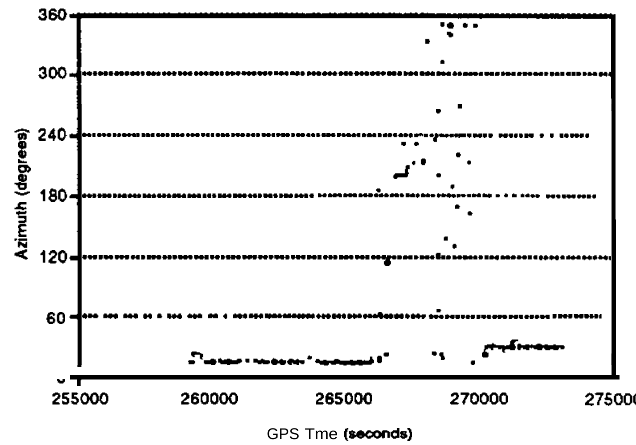

Figure 8: GPSDerived Ship Heading -June 1 Test

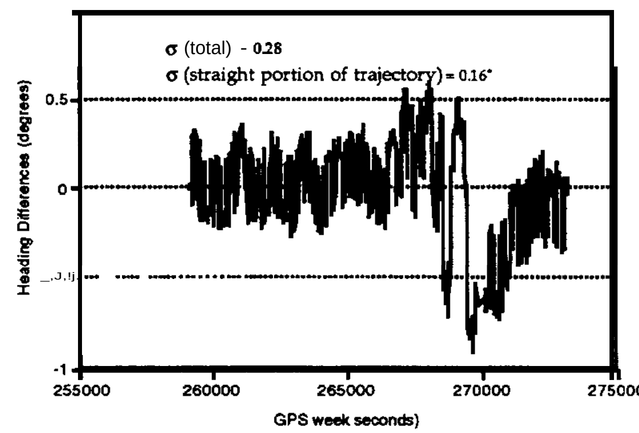

Figure 9: Heading Differences between GPS and Gyrocompass-June 1 Test

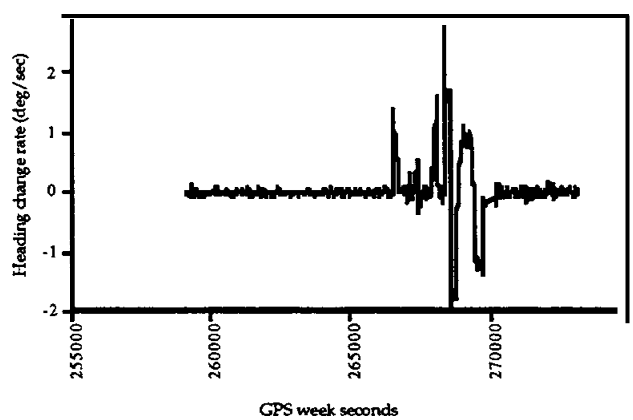

Figure 10: Heading Change Rate -June 1 Test

Comparison between the Four-Antenna System and a Two-Antenna System

When only the ship heading is required, the use of a two-antenna configuration is sufficient. In order to show the accuracy performance achievable with a two-antenna system, the ship heading parameter was estimated for both May 31 and June 1 using Antennas 1 and 2 (See Figure 1).

The heading values obtained on May 31 with the above two-antenna system are compared with the corresponding values obtained with the four-antenna system in Figure 11. The mean difference is nearly zero and the rms difference is 0.01”. Similar results were obtained on June 1. These show clearly that the two-antenna system gives virtually the same level of accuracy as the four-antenna system for heading determination.

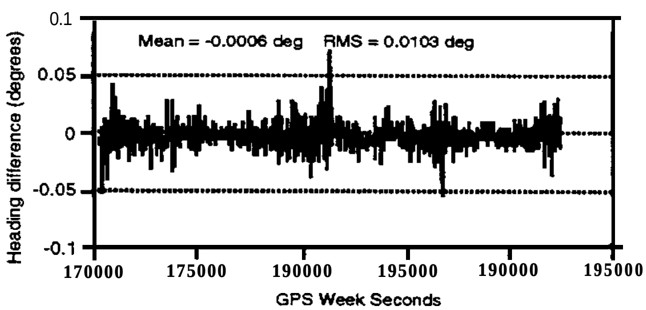

Figure **11: Heading Differences between 4-Antenna and Z-Antenna GPS Attitude System - May** *31* **Test**

CONCLUSIONS

The four-antenna non-dedicated GPS attitude determination system and post-processing software MULTINAV™ used herein have delivered a satisfactory level of performance during the sea see trials. The tracking loops of the NovAtel GPSCard™ receivers were stable during all ship manoeuvres performed and carrier phase cycle slips were few and limited to low satellites. Since MULTINAV™ resolves the carrier phase ambiguities on the fly and estimates the attitude parameters independently at each point, the GPS-derived attitude parameters are drift free, free from ship’s dynamics and are obtained with a accuracy better than 0.1’. The standard deviation of the differences between GPS-derived and gyrocompass heading measurements was, using straight trajectory segments to remove the ship’s dynamic effects on the gyrocompass, 0.16’ for each day of the trial.

The use of a two-antenna system was shown to produce a heading accuracy level similar to that obtained with the above four-antenna configuration system, independently of ship’s dynamics.

REFERENCES

CANNON, M.E., and G. LACHAPELLE (1992) Analysis of a High Performance C/A Code GPS Receiver in Kinematic Mode. Navigation, 39, 3, The Institute of Navigation, Alexandria, VA, 285-299.

LACHAPELLE, G., C. LIU, G. LU, B. TOWNSEND, M.E. CANNON, and R. HARE (1993) Precise Marine DGPS Positioning Using P Code and High Performance C/A Code Technologies. Geomatica, Canadian Institute of Geomatics, Ottawa, 47, 2, 117-128.

LU, G., M.E. CANNON, G. LACHAPELLE, and P. KIELLAND (1993) Attitude Determination in a Survey Launch Using Multi-Antenna GPS Technologies. Proceedings of National Technical Meeting, The Institute of Navigation, Alexandria, VA, 251-260.

VAN DIERENDONCK, A.J., P. FENTON, and T. FORD (1992) Theory and Performance of Narrow Correlator Spacing in a GPS Receiver. Navigation, Institute of Navigation, Alexandria, VA, 39, 3‘265-283.

WERTZ, J.R. (Ed.) (1978) Spacecraft Attitude Determination and Control. Kluwer Academic Publishers, The Netherlands.