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Application Note on Vehicle to Body Rotations

This application note gives general guidance on how to extract the vehicle's attitude with respect to the local level frame.

You can find the logs and commands mentioned in this application note in the *SPAN Technology for OEMV User Manual*. User manuals are available from our website at:

https://portal.hexagon.com/public/Novatel/assets/Documents/Manuals/om-20000104

Frame Definitions

The vehicle frame is defined as x (perpendicular to the direction of travel in the horizontal plane), y (direction of travel), and z (up).

The body frame is nominally the frame as marked on the IMU enclosure. If you do not mount the IMU with the z-axis approximately up, you must check the new IMU axis orientation that SPAN automatically uses. SPAN forces z to be up in the internal computation frame. Output attitude (in **INSPVA**, **INSATT**, and so on) is with respect to that computation frame. Refer to the **SETIMUORIENTATION** command description to see what mapping definition applies, depending on which IMU axis most closely aligns to gravity. Essentially, this means that if you do not mount the IMU with the z-axis approximately up (as marked on the enclosure); you have a new IMU frame that defines what mapping applies. This new IMU frame will not match what is marked on the IMU enclosure and will need to be determined by checking the *Full Mapping Definition* table documented with the **SETIMUORIENTATION** command. Also, in this case, begin with the new IMU frame aligned with the vehicle frame and record your vehicle to body rotations with respect to the frame SPAN will be using as the computation frame.

The output roll is the angle of rotation about the y-axis, the output pitch is about the x-axis, and the output azimuth is about the z-axis and is measured to the y-axis. Note that azimuth is positive in the clockwise direction when looking towards the origin. However, the input vehicle to body rotation about the z-axis follows the right hand rule convention.

X (east), Y (north), and Z (up) define the local level frame.

Relating the Vehicle Frame to the Body Frame:

Form the rotation matrix from the vehicle frame to the body frame using the vehicle to body frame angles measured according to the procedure described in the **VEHICLETOBODYROTATION** log as:

γ is the rotation about the z-axis
α is the rotation about the x-axis
β is the rotation about the y-axis

This direction cosine matrix (DCM) expresses mathematically the sequence of rotations as:

$$R_v^b = R_2(\beta)R_1(\alpha)R_3(\gamma)$$

Where R_3 is a rotation around the z-axis (that is, the third axis of the x, y, and z set), as in:

$$R_3(\gamma) = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1 is a rotation around the x-axis, as in:

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

R_2 is a rotation around the y-axis, as in:

$$R_2(\beta) = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}$$

These three rotations define the transformation from the vehicle frame to the body frame.

To go the other way between the frames (from the body frame to the vehicle frame), we take the transpose of this, which is:

$$R_b^v = (R_v^b)^T = R_3(\gamma)^T R_1(\alpha)^T R_2(\beta)^T = R_3(-\gamma)R_1(-\alpha)R_2(-\beta)$$

$$R_b^v = R_3(-\gamma)R_1(-\alpha)R_2(-\beta)$$

Explicitly, the elements of this matrix are:

$$R_b^v = \begin{bmatrix} \cos\gamma\cos\beta - \sin\gamma\sin\alpha\sin\beta & -\sin\gamma\cos\alpha & \cos\gamma\sin\beta + \sin\gamma\sin\alpha\cos\beta \\ \sin\gamma\cos\beta + \cos\gamma\sin\alpha\sin\beta & \cos\gamma\cos\alpha & \sin\gamma\sin\beta - \cos\gamma\sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \sin\alpha & \cos\alpha\cos\beta \end{bmatrix}$$

Relating the Body Frame to the Local Level Frame:

The body to local level matrix can be formed from the roll, pitch, azimuth (convert to yaw to form the matrix) angles found in the **INSPVA** (or **INSATT**) log, where roll is about the y-axis, pitch is about the x-axis, and azimuth/yaw is about the z-axis. The body frame (IMU axes) are nominally assumed to have Y pointing in the direction of travel, X to the right (perpendicular to the direction of travel), and Z up. That means that azimuth references from north to the IMU y-axis, and roll is about y, and pitch is about x. If you do not mount your IMU with Z approximately up, please review the *Frame Definitions* section, on *Page 1* of this document.

Using the angles in the **INSPVA** log, you can form the body frame to local-level frame-rotation matrix.

φ is roll

θ is pitch

ψ is yaw

$$R_b^l = \begin{bmatrix} \cos\psi\cos\varphi - \sin\psi\sin\theta\sin\varphi & -\sin\psi\cos\theta & \cos\psi\sin\varphi + \sin\psi\sin\theta\cos\varphi \\ \sin\psi\cos\varphi + \cos\psi\sin\theta\sin\varphi & \cos\psi\cos\theta & \sin\psi\sin\varphi - \cos\psi\sin\theta\cos\varphi \\ -\cos\theta\sin\varphi & \sin\theta & \cos\theta\cos\varphi \end{bmatrix}$$

Relating the Vehicle Frame to the Local Level Frame:

Knowing the attitude of the body frame with respect to the local level frame, from the **INSPVA** (or **INSATT**) log, and the rotational relationship between the body frame and the vehicle frame, you can solve for the vehicle attitude with respect to local level. Given the two previous matrices, a new DCM can form relating the vehicle frame to the local level frame.

$$R_v^l = R_b^v R_l^b$$

where $R_l^b = (R_b^l)^T$

We can solve the individual angles, from the local level to vehicle frame, using the numerical values of R_v^l since we know how to form the DCM. The angles from the vehicle frame to the local level frame are denoted as θ_x, θ_y, and θ_z. We give the matrix indices first as rows 1-3 and then columns 1-3.

$$\theta_x = \sin^{-1}\left[R_v^l(3,2)\right]$$

$$\theta_y = \tan^{-1}\left[\frac{-R_v^l(3,1)}{R_v^l(3,3)}\right]$$ (*In code use atan2 for proper quadrant resolution.*)

$$\theta_z = \tan^{-1}\left[\frac{-R_v^l(1,2)}{R_v^l(2,2)}\right]$$

Final Points

If you require further information, regarding the topics covered by this document, please contact:

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