APN-026
Rev 1A

## Geodetic Principles

## 1. Overview

The purpose of this document is to provide some insight into how certain geodetic principles relate to your GPS receiver. Here you can find background information relating to the geoid and ellipsoid, and how they are used to provide you with height information.

## 2. What is the Geoid?

The geoid is a level, or equipotential surface, where the gravity potential is a constant value. The gravity force vector acts perpendicular to this surface. A good example of a level surface would be a large body of water where the force of gravity acts on the water such that a constant surface is formed.

This is where the Mean Sea Level (MSL) height stems from. The surface of the ocean, after some generalization, can be considered as a good approximation to the geoid.

Geoids are generally formed by way of gravity measurements and intensive computations to formulate models. These models provide a reference from which heights can be measured.


Contrast of the Geoid model with an Ellipsoid and cross-section of the Earth's surface.
Source: https://www.usgs.gov/media/images/geoid-illustration

## 3. What is the Ellipsoid?

The ellipsoid is an approximation to the geoid. It can be derived mathematically and is generally referred to as a terrestrial reference frame. The World Geodetic System 1984 (WGS-84) is an ellipsoid that all GPS measurements reference.

Position information can be displayed in two standard ways:

1. Curvilinear Coordinates
$\phi$ - Latitude
$\lambda$ - Longitude
h - Ellipsoidal height
2. Cartesian or Earth Centered Earth Fixed (ECEF) Coordinates
x - Passing through the Greenwich Meridian
$y$ - Orthogonal to the $x$-axis (left-handed)
z - Perpendicular to the $(\mathrm{x}, \mathrm{y})$ origin; passing through the pole

## 4. How do the Ellipsoid and Geoid relate to the GPS receiver?

If you look at the height in the BESTPOSA log from your NovAtel receivers, you are looking at Mean Sea Level (MSL) heights.

MSL heights are equivalent to the geoid. As mentioned earlier, the geoid is a constant equipotential surface that has been defined by making gravity measurements all over the earth.

The height above this surface is known as an Orthometric Height and for our purposes here, we will define it as H . This H value is what NovAtel provides you with the POS and BESTPOS logs.

## 5. How is the Orthometric Height Computed?

The receiver uses a scaled-down version of the OSU89B model. This is mainly to save space, thus resulting in an undulation value for every $1 \times 3$ degrees square on the earth.

The formula to use in deriving the ellipsoidal height is:

$$
\mathrm{h}=\mathrm{H}+\mathrm{N}
$$

where
h - is your ellipsoidal height.
H - is your Mean Sea Level height.
N - is your undulation value.
The receiver uses an internal table to solve for H given the ellipsoidal height from the GPS measurements and the undulation value from the table.

Rearranging the equation and solving for the orthometric height yields:

$$
\mathrm{H}=\mathrm{h}-\mathrm{N}
$$

The undulation N is the separation between the ellipsoid and the geoid.

## Examples:

The following data is from a PwrPak7 receiver using the NovAtel roof top antenna network.

```
#BESTPOSA, USB2, 0, 28.5, FINESTEERING, 2077, 335890.000, 02000020, b1f6,
15823; SOL_COMPUTED, SINGLE, 51.15043791235, -114.03067371345, 1097.4456,
-17.0000, WGS84, 1.4238, 0.8078, 2.2276, "", 0.000, 0.000, 35, 28, 28, 28,
00, 06, 35, 33*dd90085a
```

The value, -17.0000 in the above $\log$ is known as the undulation value. The value, 1097.4456 is the MSL height computed using the onboard undulation table, therefore, to convert this to an ellipsoidal height:

$$
\begin{aligned}
\mathrm{h} & =\mathbf{1 0 9 7 . 4 4 5 6}+(\mathbf{- 1 7 . 0 0 0 0}) \\
& =1080.4456 \mathrm{~m}
\end{aligned}
$$

The ellipsoidal height at this location will be 1080.4456 meters with respect to WGS84. The same result can be achieved by using the undulation command. By providing the receiver with the following command:
undulation user 0.0
This command will set the undulation value to a user defined value of 0.0 m .

```
#BESTPOSA, USB2, 0, 25.0, FINESTEERING, 2077, 336581.000, 02000020, b1f6,
15823; SOL_COMPUTED, SINGLE, 51.15043413184, -114.03067943945, 1080.1762,
0.0000, WGS84, 1.4370, 0.8045, 2.3162, "", 0.000, 0.000, 36, 27, 27, 27, 00,
06, 35, 33*9f32c1ff
h = 1080.1762+0.0000
    =1080.1762m
```

You can see from the two ellipsoidal height computations that using the undulation command and setting the value to 0.0 is similar to applying the value to achieve ellipsoidal heights.

## 6. What if $I$ have specified a different Datum?

If you have set the datum, please note that this command specifically uses WGS84 as a reference. Therefore, you will be unable to apply the "undulation user 0.0 " command directly. By implying a different datum than WGS84, there is an additional shift applied to the coordinates that must be accounted for.

## Examples:

```
#BESTPOSA, USB2, 0, 20.0, FINESTEERING, 2077, 416417.000, 02040020, b1f6,
15823; SOL_COMPUTED, SINGLE, 51.15044240863, -114.03068619018, 1099.5250,
-17.0000, WGS84, 1.1305, 0.9535, 2.3109, "", 0.000, 0.000, 37, 28, 28, 28,
00, 06, 35, 33*9d2bb280
```

Above, in bold, you can see the undulation value and the current datum from the BESTPOSA log. The ellipsoidal height computation is similar to the previous example.

$$
\begin{aligned}
\mathrm{h} & =1099.5250+(-17.0000) \\
& =1082.525 \mathrm{~m}
\end{aligned}
$$

The next $\log$ is the position determined relative to the TOKYO datum.
To set the datum relative to TOKYO, enter the command in your terminal or NovAtel Connect "DATUM TOKYO"

```
#BESTPOSA, USB2, 0, 38.0, FINESTEERING, 2077, 421492.000, 02000020, b1f6,
15823; SOL_COMPUTED, SINGLE, 51.14340091991, -114.02621028017, 1097.7197,
408.3758, TOKYO, 1.2049, 0.8450, 1.8695, "", 0.000, 0.000, 37, 30, 30, 30,
00, 06, 35, 33*fd6f00c2
```

From this, it is evident that the MSL height is relatively similar to its WGS84 counterpart when factoring in that the data is from single point measurements. The ellipsoidal height is as follows:

$$
\begin{aligned}
\mathrm{h} & =1097.7197+(408.3758) \\
& =1506.0955 \mathrm{~m}
\end{aligned}
$$

Since the position was computed using the TOKYO datum that is based upon the Bessel 1841 ellipsoid, you can see a much different ellipsoidal height value.

```
#BESTPOSA, USB2, 0, 26.5, FINESTEERING, 2077, 423337.000, 02000020, b1f6,
15823; SOL_COMPUTED, SINGLE, 51.14339877743, -114.02621320190, 1079.7341,
425.3758, TOKYO, 1.1985, 0.7969, 1.8567, "", 0.000, 0.000, 34, 28, 28, 28,
00, 06, 35, 33*0cf85347
```

The BESTPOS log shows the result of applying the "undulation user 0.0 " command. When the undulation value is set to 0.0 and a datum other than WGS84 is used, you cannot achieve ellipsoidal heights as shown in the above example.

In this case, $\mathrm{N}=-17.0000 \mathrm{~m}$ at the current location. Setting the undulation to 0.0 removes the value from the MSL computation thus leaving 1079.7341 m . This does not provide the correct ellipsoidal height because we are dealing with the TOKYO datum rather than with WGS84.

## 7. How can I get the undulation values for pre- or post-mission analysis?

If for any reason you require the undulation values that the receiver is applying to the position logs, you can do the following:

- Connect a NovAtel receiver to your PC and issue a FIX POSITION command. See our documentation portal for more information on the FIX POSITION command here: https://docs.novatel.com/OEM7/Content/Commands/FIX.htm
- Enter the latitude, longitude and height for the general location of the survey.
- Log BESTPOSA and retrieve an undulation value.

This does not require that the antenna be set up unless you do not know your general location. The undulation value is read directly from the table based on the location input to the receiver.

If you require more precise undulation values, then you should refer to an undulation data table with respect to WGS84. This data will provide an undulation value for the survey area, which can be applied to the receiver using the UNDULATION command.

### 7.1 Where can I find more precise geoid data?

You can find high precision geoid data from the following sources:
https://www.ngs.noaa.gov/GEOID/
https://www.usgs.gov/products/data-and-tools/science-datasets

## 8. What does the FIX POSITION command require in terms of height?

Fixing the position using MSL heights, allows you to enter the latitude, longitude and height based solely on how well it is known. Therefore, the significance of fixing the position using MSL is that the base station coordinates are generally known from some method of survey. Whether through conventional survey techniques or using GPS, the coordinates of the base station need to be known. In most instances, the height is known with respect to a specific datum.

If you have known coordinates in one of the supported datums, all you need to do is set the appropriate datum and fix the position. This provides a convenient method of not directly needing the undulation value or converting non-WGS84 coordinates when fixing the position.

## 9. What about post-processing?

If you are interested in a post-processing package for logging data, processing it and plotting the course, we recommend using WayPoint products.

NovAtel Waypoint Products Group offers GNSS post-processing software packages with a Graphical User Interface (GUI). These software packages include:

- GrafNet (a static baseline processor adjustment package)
- GrafNav (a static/kinematic baseline processor)
- Inertial Explorer (a GNSS+IMU post-processing software)

For more information on how these software work, read the Waypoint Software User Manual here: https://docs.novatel.com/Waypoint/Content/Home.htm

It is important to keep in mind that Waypoint Products Group does not derive any of the geoid values provided with the software. Rather, the data has been acquired from various international or government agencies and been converted into a format (WPG) readable by Waypoint's software.
10. Customer Service Contact Information

If you require any further information regarding the topics covered within this application note, please contact NovAtel Support:

Email: support@novatel.com
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